Circular Motion 2

① a) $v = \frac{2\pi r}{T} = \frac{2\pi(50)}{14.3} = 21.969 \text{ m/s}$

$a = \frac{v^2}{r} = \frac{21.969^2}{50} = \boxed{9.65 \text{ m/s}^2 \text{ [TTC]}}$

b) $\Sigma F = ma = (615)(9.65) = \boxed{5936.5 \text{ N [TTC]}}$

② a) $f = 1 \text{ Hz}$

$T = \frac{1}{f} = \frac{1}{1} = 1 \text{ s}$

$v = \frac{2\pi r}{T} = \frac{2\pi(1.3)}{1} = 8.168 \text{ m/s}$

$a = \frac{v^2}{r} = \frac{8.168^2}{1.3} = \boxed{51.3 \text{ m/s}^2 \text{ [TTC]}}$

b) $\Sigma F = T$

$ma = T$

$T = (7)(51.3) = \boxed{359 \text{ N}}$

(3) a) $f = 1\ Hz$

$T = \frac{1}{f} = \frac{1}{1} = 1\ s$

$v = \frac{2\pi r}{T} = \frac{2\pi(0.8)}{1} = 5.027\ m/s$

$\Sigma F = \frac{mv^2}{r} = \frac{(0.2)(5.027)^2}{(0.8)} = \boxed{6.32\ N\ [TTC]}$

b) $f = 2\ Hz$

$T = \frac{1}{f} = \frac{1}{2} = 0.5\ s$

$v = \frac{2\pi r}{T} = \frac{2\pi(0.8)}{(0.5)} = 10.053\ m/s$

$\Sigma F = \frac{mv^2}{r} = \frac{(0.2)(10.053)^2}{(0.8)} = \boxed{25.3\ N\ [TTC]}$

c) 4:1 ratio because doubling the frequency doubles the speed. Since $F \propto v^2$ that means F will quadruple when v is doubled.

④ a) TTC (because it moves in a circle at uniform speed)

b) $f = 33.\overline{3}\ \text{rpm} \times \frac{1\ \text{min}}{60\ \text{s}} = 0.\overline{5}\ \text{Hz}$

$T = \frac{1}{f} = \frac{1}{0.\overline{5}} = 1.8\ \text{s}$

5 cm $v = \frac{2\pi r}{T} = \frac{2\pi(0.05)}{1.8} = 0.175\ \text{m/s}$

$a = \frac{v^2}{r} = \frac{0.175^2}{0.05} = \boxed{0.61\ \text{m/s}^2\ [\text{TTC}]}$

10 cm $v = \frac{2\pi(0.1)}{1.8} = 0.349\ \text{m/s}$

$a = \frac{v^2}{r} = \frac{0.349^2}{0.1} = \boxed{1.22\ \text{m/s}^2\ [\text{TTC}]}$

15 cm $v = \frac{2\pi(0.15)}{1.8} = 0.524\ \text{m/s}$

$a = \frac{v^2}{r} = \frac{0.524^2}{0.15} = \boxed{1.83\ \text{m/s}^2\ [\text{TTC}]}$

c) friction

d) 15 cm because the faster it goes, the more friction is required, but the amount of available friction is limited.

⑤ a) $a = \frac{v^2}{r} = \frac{2.10^2}{0.153} = \boxed{26\,405\,882 \text{ m/s}^2 \text{ [TTC]}}$

b) $\Sigma F = ma$

$= (0.001)(26\,405\,882)$

$\Sigma F = \boxed{26\,405.9 \text{ N [TTC]}}$

c) $v = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v} = \frac{2\pi(0.153)}{2.10} = \boxed{4.78 \times 10^{-4} \text{ s}}$

⑥ $f = 3 \text{ Hz}$

$T = \frac{1}{f} = \frac{1}{3} = 0.\overline{3} \text{ s}$

$v = \frac{2\pi r}{T} = \frac{2\pi(1)}{0.\overline{3}} = 18.850 \text{ m/s}$

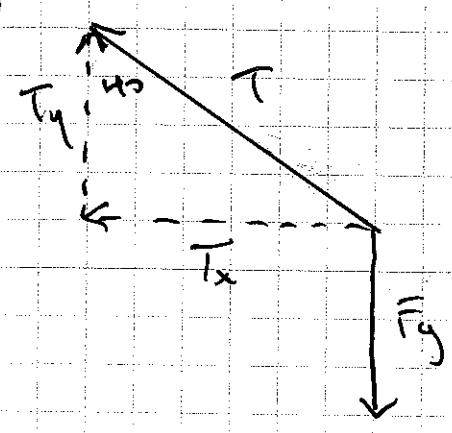
$\Sigma F = \frac{mv^2}{r} = \frac{(0.208)(18.850)^2}{1} = \boxed{73.9 \text{ N}}$

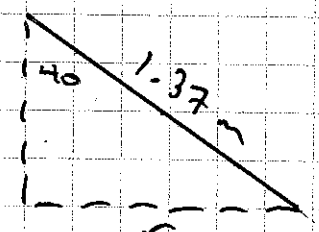
⑦ $T = \frac{1}{f} = \frac{1}{0.811} = 1.233\ s$

$v = \frac{2\pi r}{T} = \frac{2\pi(0.452)}{1.233} = 2.303\ m/s$

$T = \Sigma F = \frac{mv^2}{r} = \frac{(0.436)(2.308)^2}{0.452} = \boxed{5.12\ N\ [TTC]}$

⑧





$\sin 40 = \frac{r}{1.37}$

$r = 1.37 \sin 40 = 0.881\ m$

a) $T_y = F_y = mg = (0.255)(9.8) = 2.499\ N$

$\tan 40 = \frac{T_x}{T_y}$

$\Sigma F = T_x = T_y \tan 40 = 2.499 \tan 40 = 2.097\ N$

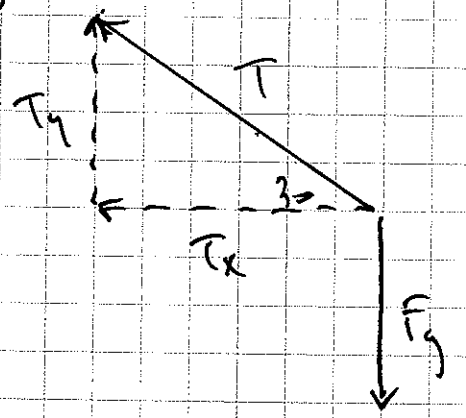
$\Sigma F = \frac{mv^2}{r}$

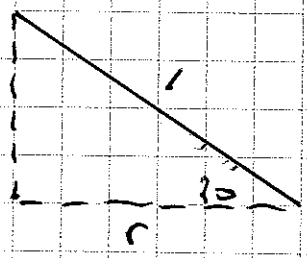
$v = \sqrt{\frac{\Sigma F r}{m}} = \sqrt{\frac{(2.097)(0.881)}{(0.255)}} = \boxed{2.69\ m/s}$

(8) b) $\cos 40 = \frac{T_y}{T}$

$$T = \frac{T_y}{\cos 40} = \frac{2.499}{\cos 40} = \boxed{3.26\ N}$$

(9)





$\cos 30 = \frac{r}{1}$

$r = 0.866\ m$

a) $T_y = F_g = mg = (2)(9.8) = 19.6\ N$

$\sin 30 = \frac{T_y}{T}$

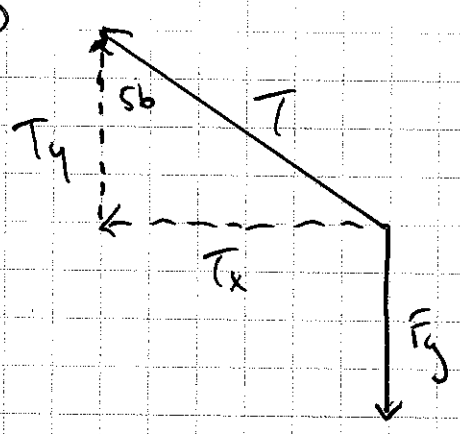
$$T = \frac{T_y}{\sin 30} = \frac{19.6}{\sin 30} = \boxed{39.2\ N}$$

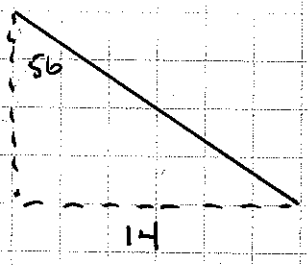
b) $\tan 30 = \frac{T_y}{T_x}$

$$\Sigma F = T_x = \frac{T_y}{\tan 30} = \frac{19.6}{\tan 30} = 33.948\ N$$

$$v = \sqrt{\frac{\Sigma F \cdot r}{m}} = \sqrt{\frac{(33.948)(0.866)}{2}} = \boxed{3.83\ m/s}$$

(10)





$T_y = F_g = mg$

$\tan 56 = \frac{T_x}{T_y}$ $\qquad T_x = T_y \tan 56 = mg \tan 56$

$\Sigma F = \frac{mv^2}{r}$ $\qquad \Sigma F = T_x$

$T_x = \frac{mv^2}{r}$

$mg \tan 56 = \frac{mv^2}{r}$

$v = \sqrt{gr \tan 56}$

$v = \sqrt{(9.8)(14)\tan 56}$

$v = \boxed{14.3 \text{ m/s}}$

(11) a) $f = 1.1\ Hz$

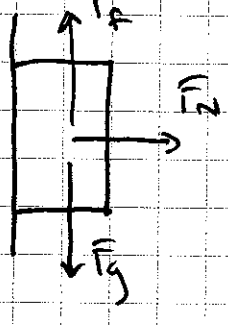
$$T = \frac{1}{f} = \frac{1}{1.1} = 0.909\ s$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(2)}{0.909} = \boxed{13.823\ m/s}$$

b) $$a = \frac{v^2}{r} = \frac{13.823^2}{2} = \boxed{95.538\ m/s^2\ [TTC]}$$

c) The wall pressing against the person's back.

d)



$$\vec{F}_N = \Sigma \vec{F} = \frac{mv^2}{r}$$

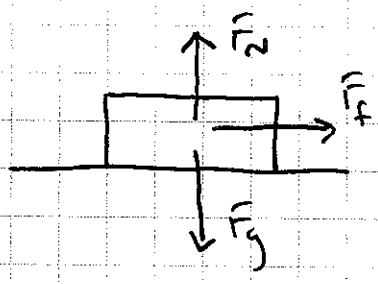
$$\begin{cases} \vec{F}_f = \vec{F}_g = mg \\ \vec{F}_f = \mu \cdot \vec{F}_N \end{cases}$$

$$\mu \cdot \vec{F}_N = mg$$

$$\mu \cdot \frac{mv^2}{r} = mg$$

$$\mu = \frac{rg}{v^2} = \frac{(2)(9.8)}{(13.823)^2} = \boxed{0.10}$$

(12)



$$\vec{F}_N = \vec{F}_g = mg$$

$$F_f = \mu \cdot \vec{F}_N = \mu \cdot mg$$

$$\Sigma \vec{F} = \vec{F}_f$$

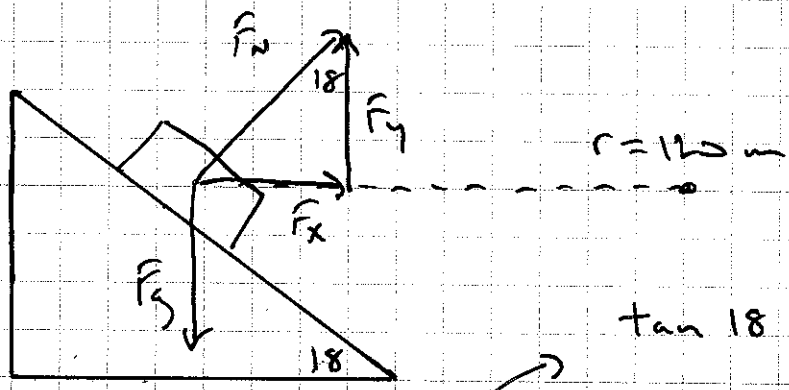
$$\frac{mv^2}{r} = \mu mg$$

$$v = \sqrt{\mu \cdot r \cdot g}$$

$$= \sqrt{(0.3)(80)(9.8)}$$

$$v = \boxed{15.3 \text{ m/s}}$$

(13)



$$F_y = \vec{F}_g = mg$$

$$\vec{F}_x = \Sigma \vec{F} = \frac{mv^2}{r}$$

$$\tan 18 = \frac{\vec{F}_x}{F_y}$$

$$= \frac{\frac{mv^2}{r}}{mg}$$

$$= \frac{mv^2}{r} \cdot \frac{1}{mg}$$

$$\tan 18 = \frac{v^2}{rg}$$

$$v = \sqrt{rg \tan 18} = \sqrt{120(9.8)\tan 18} = \boxed{19.5 \text{ m/s}}$$

(14) Same work as for # 13.

$$v = \sqrt{rg\tan\theta}$$

$$v^2 = rg\tan\theta$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$= \tan^{-1}\left[\frac{25^2}{(150)(9.8)}\right]$$

$$\theta = \boxed{23^\circ}$$

(15) Same work as for # 13.

$$v = \sqrt{rg\tan\theta}$$

$$= \sqrt{(100)(9.8)\tan 68}$$

$$v = \boxed{49.3 \text{ m/s}}$$

(16) Same work as for # 14

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$= \tan^{-1}\left[\frac{(17.306)^2}{(77)(9.8)}\right]$$

$$\theta = \boxed{21.6^\circ}$$